Binary Numbers

Binary
Octal
Hexadecimal

Binary Numbers

- COUNTING SYSTEMS UNLIMITED . . . Since you have been using the 10 different digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 all your life, you may wonder how it is possible to count and do arithmetic without using all 10. Actually, there is no advantage in using 10 counting digits rather than, say, 8, 12, or 16. The 10-digit system (called the decimal system, since the word "decimal" means "based on 10") probably came into universal use because man first started to count by using his fingers, and there happen to be 10 of them.

- To see how to count by using other than 10 digits, notice how we count in the ordinary decimal system. We represent a number higher than 9, the highest digit, by a combination of two or more digits. The number next after 9 is 10, and then 11, etc. After we reach 99, the highest number that can be written with two digits, we start using three digits. The number next after 99 is 100, and then comes 101, etc.

- Now let's try counting in the octal system. "Octal" means "based on eight"; that is, we use only the eight digits 0, 1, 2, 3, 4, 5, 6, and 7. The digits 8 and 9 are not used. So now what do we do after we have counted to 7? Since we have used up all the symbols we are permitted to use, we write 10 as the next number and then comes 11 and so on up to 17. After 17 comes 20.

Decimal	Octal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11
10	12
11	13
12	14
13	15
14	16
15	17
16	20
17	21
18	22
19	23

Binary Numbers

- THE NATURAL BINARY SYSTEM ... Now that you have seen how it is possible to count in numbering systems other than the decimal system, we shall consider the system of most interest in electronics. That is the binary system, which uses only the two digits 0 and 1.

- We can count in the binary system by using the plan explained in the preceding topic for counting in other systems. The first number in counting is 1, of course. Since we can use no digit higher than 1, we must go to two digits and write 10 for the second binary number. Then comes 11, and after that we must go to three digits and write 100.

- Binary numbers as written in the table form the *natural* binary numbering system. It is called natural because it follows the general counting method used in the decimal, octal, and other numbering systems. As you will see later in the lesson, the natural binary system is only one of a number of methods for representing numbers by using only the digits 0 and 1.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111
16	20	10000
17	21	10001
18	22	10010
19	23	10011

Binary Numbers

CONVERSION BETWEEN OCTAL AND BINARY SYSTEMS ... As you have no doubt observed by this time, writing out and reading numbers in natural binary form is quite a nuisance because of the large number of digits involved. Since it is easy to convert natural binary numbers into octal numbers, it is practical to write or machine print out natural binary numbers as octal numbers for ease in handling. A couple of examples will show you how the conversions are made.

- EXAMPLE ... Convert binary number 1011010 to the octal equivalent.
- SOLUTION . . . The first step is to rewrite the number with the digits grouped in threes:

001 011 010

- Note that two zeros were placed in front of the first digit 1 in order to make every group complete.
- Next, write the decimal equivalent over each group of three:

1 3 2
001 011 010

- The octal equivalent of binary 1011010 is 132.

Binary Numbers

- The *hexadecimal system*, or Hex, uses base 16, therefore there are 16 possible digit symbols. The hexadecimal system groups binary number by 4's and from 0 to 9 it is the same as a decimal number equivalent in binary form. This means 0000 is 0, 0001 is 1, 0010 is 2 and so on to 1001 being 9, but then from 1010 to 1111 of binary the hexadecimal uses letters from A to F and then when it reaches the value of 16 it becomes 10 because the two groups of four binary numbers are 0001 0000. When taken as a binary number it is 0001 0000 while the decimal number is 16 and the hexadecimal number is 10. Therefore an 8 bit binary number (byte) is divided into two groups of four bits each. The chart in the next slide shows all of this.

Decimal	Octal	Binary	Hex
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	10	1000	8
9	11	1001	9
10	12	1010	A
11	13	1011	B
12	14	1100	C
13	15	1101	D
14	16	1110	E
15	17	1111	F
16	20	0001 0000	10
17	21	0001 0001	11
18	22	0001 0010	12
19	23	0001 0011	13

Binary Numbers

CONVERSION BETWEEN HEXADECIMAL AND BINARY SYSTEMS ... As you have no doubt observed by this time, writing out and reading numbers in natural binary form is quite a nuisance because of the large number of digits involved. Since it is easy to convert natural binary numbers into hexadecimal numbers, it is practical to write or machine print out natural binary numbers as hexadecimal numbers for ease in handling. A couple of examples will show you how the conversions are made.

- EXAMPLE ... Convert binary number 1011010 to the hexadecimal equivalent.
- SOLUTION . . . The first step is to rewrite the number with the digits grouped in fours:

0101 1010

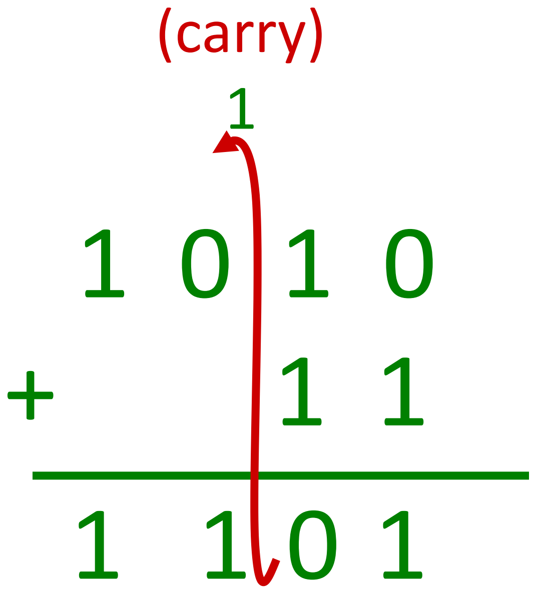
- Note the zero were placed in front of the first digit 1 in order to make the two groups complete.
- Next, write the decimal equivalent over each group:

5 A
0101 1010

- The hexadecimal equivalent of binary 1011010 is 5A.

Binary Addition

- Conceptually similar to decimal addition
- *Example:* Add the binary numbers 1010 and 11



(carry)

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & 0 & 1 & 0 \\ + & & & 1 & 1 \\ \hline & 1 & 1 & 0 & 1 \end{array}$$

Binary Subtraction

Example:

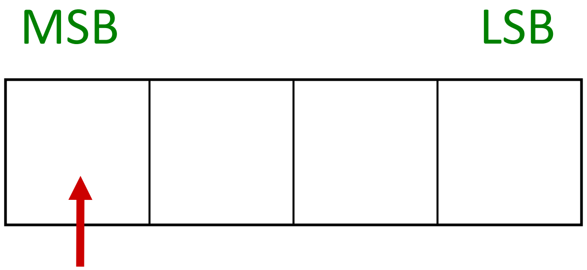
Subtract binary number 101 from 1011

(borrow)

$$\begin{array}{ccccc} & 0 & 1 & & \\ & 1 & \cancel{0} & 1 & 1 \\ - & & 1 & 0 & 1 \\ \hline & 0 & 1 & 1 & 0 \end{array}$$

2s Complement Notation

- 2s complement representation - widely used in microprocessors.
- Represents *sign* and *magnitude*



Sign bit (0 = + ; 1 = -)

Decimal:	+7	+4	+1	0	-1	-4	-7
2s Complement:	0111	0100	0001	0000	1111	1100	1001

2s Complement - Conversions

- Converting positive numbers to 2s complement:
  - Same as converting to binary
- Converting negative numbers to 2s complement:

Decimal to 2s Complement

- 4 (decimal)

↓ Convert decimal to binary

0 1 0 0

↓ 1s complement

1 0 1 1

↓ Add 1

- 4 = 1 1 0 0 (2s Complement)

2s Complement to Binary

1 1 0 0 (2s C)

↓ 1s complement

0 0 1 1

↓ Add 1

0 1 0 0 (Binary)

Adding/Subtracting in 2s Complement

2s complement notation makes it possible to add and subtract signed numbers

(Decimal)	2s Complement	
(- 1)	1 1 1 1	
+ (- 2)	+ 1 1 1 0	
(- 3)	$\boxed{1}$ 1 1 0 1	2s complement
	Discard	
(+1)	0 0 0 1	
+ (- 3)	+ 1 1 0 1	
(- 2)	1 1 1 0	2s complement

Practical Suggestion for Binary Math

- Use a scientific calculator.
- Most scientific calculators have DEC, BIN, OCT, and HEX modes and can either convert between codes or perform arithmetic in different number systems.
- Most scientific calculators also have other functions that are valuable in digital electronics such as AND, OR, NOT, XOR, and XNOR logic functions.