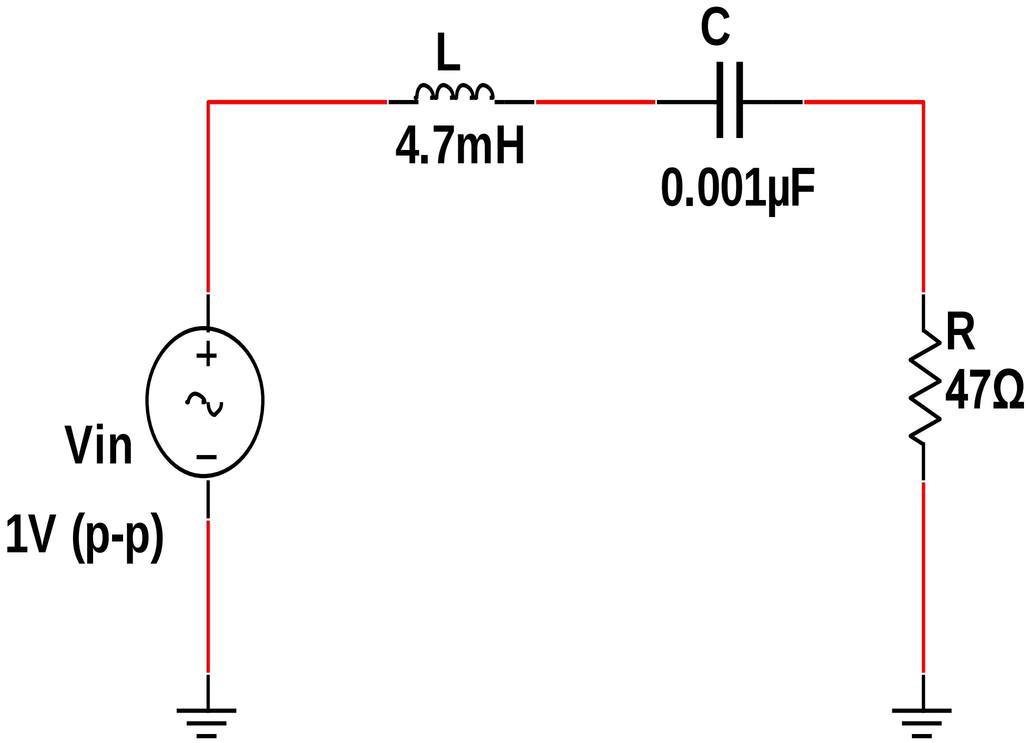
Series and Parallel Resonance

Series Resonant circuit



f_r = ?

X_C = ?

X_L = ?

I_T = ?

V_C = ?

V_L = ?

Q = ?

Δf = ?

Θ = ?

Find f_r

- $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{(4.7\times10^{-3})(0.001\times10^{-6})}} = \frac{1}{6.28\sqrt{4.7\times10^{-12}}} = \frac{1}{1.362\times10^{-5}}$ = 73.412kHz

Find reactances

- $X_L = 2\pi fL = (6.28)(73.142X10^3)(4.7X10^{-6}) = 2.168k\Omega$
- $X_C = \frac{1}{2\pi f C} = \frac{1}{(6.28)(73.142\times10^3)(0.001\times10^{-6})} = \frac{1}{4.613\times10^{-4}} = 2.168k\Omega$

Since X_C and X_L are equal, along with being 180° out of phase, the net reactance is zero which makes the total impedance equal to the resistor $\therefore Z_T = R$

Find total current and voltages

- $I_T = \frac{V_{in}}{Z_T} = \frac{V_{in}}{R} = \frac{0.3535}{47}$ = 7.521mA

Since this is a series circuit, the current found for the total will also be the current flowing through the reactive components.

- $V_C = (X_C)(I_T)$ = (2.168kΩ)(7.521mA) = 16.306V
- $V_L = (X_L)(I_T)$ = (2.168kΩ)(7.521mA) = 16.306V

As you can see, the resonant circuit appears to amplify the voltages.

Find the Q of the circuit

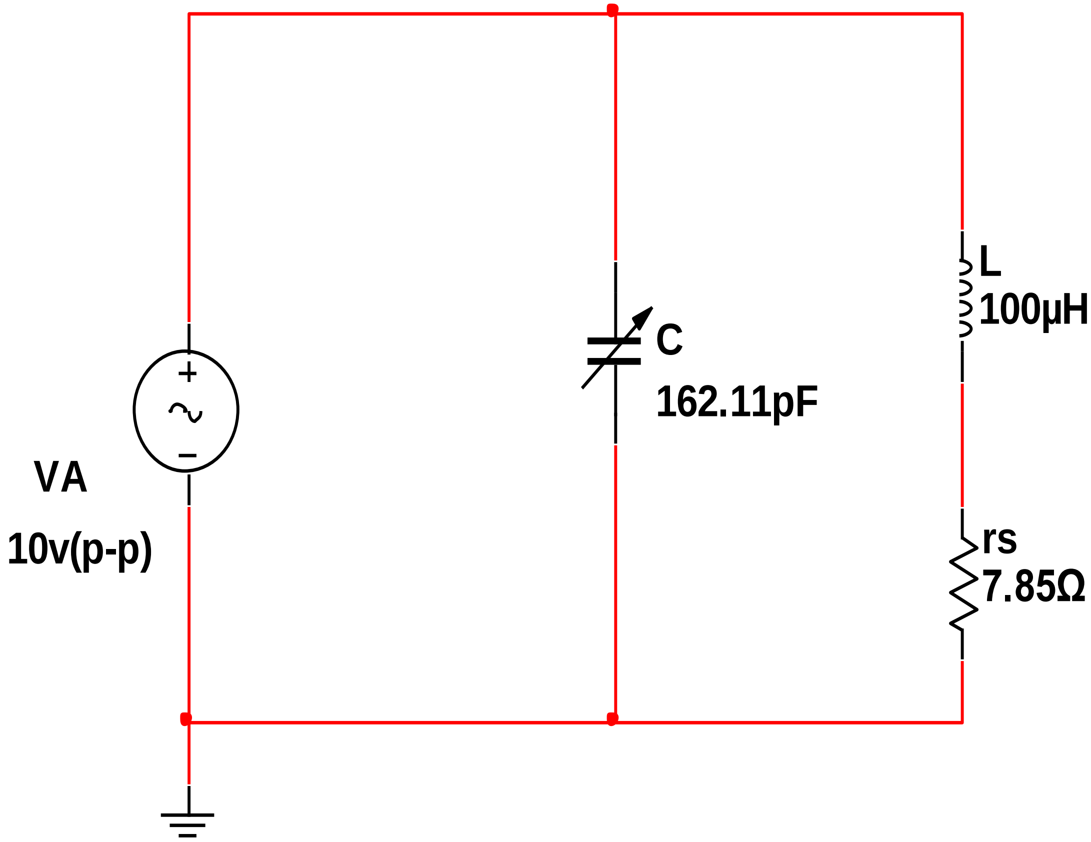
- Q = $\frac{X_L}{r_s}$ ➔ Since there is no value given for a resistance of the coil, we have to use the only resistance in the circuit to find this value

$$\therefore Q = \frac{X_L}{R} = \frac{2.168k\Omega}{47\Omega} = 46.128$$

Solve for Bandwidth and Cutoff frequencies

- $\Delta f = f_2 - f_1 = \frac{f_r}{Q} = \frac{73.142kHz}{46.128}$ = 1.592kHz
  ➔This means the frequency will vary ±796Hz $\left(\frac{\Delta f}{2}\right)$. The entire range is also known as Bandwidth.
- $f_2 = f_r + \frac{\Delta f}{2}$ = 73.412kHz + 796Hz = 74.208kHz
- $f_1 = f_r - \frac{\Delta f}{2}$ = 73.412kHz - 796Hz = 72.616kHz
- θ = 0° since X_L and X_C are canceling, which means at resonance the circuit is purely resistive.

Parallel Resonant circuit



f_r = ?

X_L = ?

X_C = ?

I_C = ?

I_L = ?

Q = ?

Z_{eq} = ?

I_T = ?

Δf = ?

Solve for f_r

- $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{(100\times10^{-6})(162.11\times10^{-12})}} = \frac{1}{6.28\sqrt{1.621\times10^{-14}}} = \frac{1}{799\times10^{-9}} = 1.25\text{MHz}$

Find the reactances

- $X_L = 2\pi fL = (6.28)(1.25X10^6)(100X10^{-6}) = 785.394\Omega$
- $X_C = \frac{1}{2\pi f C} = \frac{1}{(6.28)(1.25\times10^{6})(162.11\times10^{-12})} = \frac{1}{1.273\times10^{-3}} = 785.417\Omega$

Since this is a parallel circuit, we presume the applied voltage will be across each reactive component.

Find branch currents and the equivalent impedance

- $I_C = \frac{V_A}{X_c} = \frac{10}{785.417}$ = 12.732mA
- $I_L = \frac{V_A}{X_L} = \frac{10}{785.398}$ = 12.732mA
- $Q = \frac{X_L}{r_s} = \frac{785.398}{7.85} = 100.051 \cong 100$
- $Z_{eq} = QX_L$ = (100.051)(785.398) = 78.58kΩ ➔ Since this is the only way we are going to get the total impedance, we now need to use it to find the total current.

Find total current

- $I_T = \frac{V_A}{Z_{eq}} = \frac{10}{78.58k\Omega}$ = 127.259µA

Again, we can see the magnification of the current due to resonance.

Solve for Bandwidth and Cutoff frequencies

- $\Delta f = f_2 - f_1 = \frac{f_r}{Q} = \frac{1.25MHz}{100.051}$ = 12.494kHz
  ➔This means the frequency will vary ±6.247kHz $\left(\frac{\Delta f}{2}\right)$
- $f_2 = f_r + \frac{\Delta f}{2}$ = 1.25MHz + 6.247kHz = 1.256MHz
- $f_1 = f_r - \frac{\Delta f}{2}$ = 1.25MKz – 6.247kHz = 1.243MHz