Network Analysis IV

Mesh Equations – Two Loops
Using the method of mesh currents, solve for all the unknown values of voltage and current in the figure shown. To do this, we will complete steps a through m.
Listing of steps

a. Identify the components through which the mesh current $I_A$ flows.
b. Identify the components through which the mesh current $I_B$ flows.
c. Which component has opposing mesh currents (if any)?
d. Write the mesh equation for mesh A.

e. Write the mesh equation for mesh B.

f. Solve for the currents $I_A$ and $I_B$ using any of the methods for the solution of simultaneous equations.
g. Determine the values of currents $I_1$, $I_2$ and $I_3$.

h. Are the assumed directions of mesh A and mesh B currents correct? How do you know?

i. What is the direction of current $I_3$ through $R_3$?

j. Solve for the voltage drops $V_{R1}$, $V_{R2}$ and $V_{R3}$. 
k. Using the final solutions for $V_{R1}$, $V_{R2}$ and $V_{R3}$, write a KVL equation for the loop ACDBA going clockwise from point A.

l. Using the final solutions for $V_{R1}$, $V_{R2}$ and $V_{R3}$, write a KVL equation for the loop EFDCE going clockwise from point E.

m. Using the final solutions (and directions) for $I_1$, $I_2$ and $I_3$, write a KCL equation for the currents at point C.
Step a solution

a. Identify the components through which the mesh current $I_A$ flows.

- $I_A$ flows through $V_1$, $R_1$ and $R_3$ (not necessarily in that order).
Step b solution

b. Identify the components through which the mesh current $I_B$ flows.

- $I_B$ flows through $V_2$, $R_2$ and $R_3$ (not necessarily in that order).
c. Which component has opposing mesh currents (if any)?

- The component with the opposing mesh currents is $R_3$ (down through $R_3$ from $I_A$ and up through $R_3$ from $I_B$).
d. Write the mesh equation for mesh A.

\[ 20I_A - 10I_B = -40V \]
e. Write the mesh equation for mesh B.

- \(-10I_A + 25I_B = -20V\)
Step f solution

Original equations:

- Step d:
  \[ 20I_A - 10I_B = -40V \]

- Step e:
  \[ -10I_A + 25I_B = -20V \]

f. Solve for the currents \( I_A \) and \( I_B \) using any of the methods for the solution of simultaneous equations.

- Step 1 ➔ Divide equation from step d by 2 (i.e. make values of \( I_A \) the same)
  \[ 10I_A - 5I_B = -20V \]
Step f solution (cont.)

• Step 2 ➔ Add equations

\[
\begin{align*}
10I_A - 5I_B &= -20V \\
-10I_A + 25I_B &= -20V \\
20I_B &= -40V
\end{align*}
\]
Step f solution (cont.)

• Step 3 ➞ Solve for $I_B$ by dividing multiplier

\[
\frac{20I_B}{20} = \frac{-40}{20} \quad \therefore \quad I_B = -2A
\]
Step f solution (cont.)

- Step 4 ➔ Substitute $I_B$ into either equation to solve for $I_A$ (we will use one from step d)

$$20I_A - 10(-2A) = -40 \implies 20I_A + 20 = -40$$
Step f solution (cont.)

• Step 5 ➔ Move all the known to the right side, which will leave our unknown on the left

\[ 20I_A = -60 \]

• Step 6 ➔ Divide multiplier to solve for \( I_A \)

\[
\frac{20I_A}{20} = \frac{-60}{20} \quad \therefore \quad I_A = -3A
\]
Step g solution

g. Determine the values of currents $I_1$, $I_2$ and $I_3$.

- $I_1 = I_A = -3\,\text{A}$
- $I_2 = I_B = -2\,\text{A}$
- $I_3 = I_B - I_A = -2\,\text{A} - (-3\,\text{A}) = -2\,\text{A} + 3\,\text{A} = 1\,\text{A}$
Step h solution

h. Are the assumed directions of mesh A and mesh B currents correct? How do you know?

- The assumed currents are not correct as they are negative in value. The more appropriate statement would be a reversal of the current directions.
Step i solution

i. What is the direction of current $I_3$ through $R_3$?

• The overall direction of $I_3$ is up through $R_3$ (counterclockwise for mesh A and clockwise for mesh B)
Step j solution

- Solve for the voltage drops $V_{R1}$, $V_{R2}$ and $V_{R3}$.

- $V_{R1} = I_1 R_1 = 3A(10\Omega) = 30V$
- $V_{R2} = I_2 R_2 = 2A(15\Omega) = 30V$
- $V_{R3} = I_3 R_3 = 1A(10\Omega) = 10V$
k. Using the final solutions for $V_{R1}$, $V_{R2}$ and $V_{R3}$, write a KVL equation for the loop ACDBA going clockwise from point A.

$$30V + 10V - 40V = 0$$
Step I solution

I. Using the final solutions for $V_{R1}$, $V_{R2}$ and $V_{R3}$, write a KVL equation for the loop EFDCE going clockwise from point E.

$$-20V - 10V + 30V = 0$$
Step m solution

• Using the final solutions (and directions) for $I_1$, $I_2$ and $I_3$, write a KCL equation for the currents at point C.

• $I_3 + I_2 = I_1$; this translates into the KCL equation

$$I_3 + I_2 - I_1 = 0 \implies 1A + 2A - 3A = 0$$

($I_3$ and $I_2$ flow into node C, while $I_1$ flows out of node C).
The circuit with directions and voltage measurements

- V1: 40 V
- V2: 20 V
- R1: 10Ω
- R2: 15Ω
- R3: 10Ω
The End