Algebra Part 6

Factoring Monomials and Polynomials
Factoring (definition)

• To factor a polynomial is to find an equivalent expression that when multiplied, equals the original expression.

• To factor a monomial, we need to find two monomials whose product is equivalent to the original monomial.

• i.e. $15x^3 = (3\cdot 5) (x\cdot x^2)$
Factoring When Terms Have a Common Factor

• Example One
  \[ 5x^2 + 15 \]
  \[ 5(x^2 + 3) \]

• Example Two
  \[ 15x^5 - 12x^4 + 27x^3 - 3x^2 \]
  \[ 3x^2 (3x^3 - 4x^2 + 9x - 1) \]
Factoring by Grouping

- Algebraic expressions can contain a common factor with two or more terms.

- Example One
  \[ x^2(x+1) + 2(x+1) \]
  \[ (x + 1) (x^2 + 2) \]
Factoring by grouping (cont)

- Example Two

\[5x^3 - x^2 + 15x - 3\]

\[5x^3 - x^2 = x^2(5x - 1)\] and

\[15x - 3 = 3(5x - 1)\] therefore

\[5x^3 - x^2 + 15x - 3 = x^2(5x - 1) + 3(5x - 1)\] therefore

\[(x^2 + 3)(5x - 1)\]
Let’s try a few - (factoring)

- $y(y - 2) + 7(y - 2)$
- $(y + 7)(y - 2)$

- $y^2(y + 8) + (y + 8)$
- $(y^2 + 1)(y + 8)$
Let’s try a few - Factoring by grouping

- $9x^3 - 12x^2 + 3x - 4$
- $3x^2 (3x - 4) + 1(3x - 4)$
- $(3x^2 + 1) (3x - 4)$
- $7x^3 + 2x^2 - 14x - 4$
- $7x^3 - 14x + 2x^2 - 4$ (notice the re-ordering!!)
- $7x(x^2 - 2) + 2 (x^2 - 2)$ (now there are like factors)
- $(7x + 2) (x^2 - 2)$
Let’s try simplifying

- \((x + 3)(x + 5)\)
- \(x^2 + 5x + 3x + 15\)
- \(x^2 + 8x + 15\)
- \((2x + 5)(3x - 4)\)
- \(6x^2 - 8x + 15x - 20\)
- \(6x^2 + 7x - 20\)
More simplification

- $(3t - 5)^2$
- $(3t - 5) (3t - 5)$
- $9t^2 - 15t - 15t + 25$
- $9t^2 - 30t + 25$
Just one more

- $(2t - 9)^2$
- $(2t - 9)(2t - 9)$
- $4t^2 - 18t - 18t + 81$
- $4t^2 - 36t + 81$